Problem 4

Solve the inequality $|x-1|-|x-3| \geq 5$.

Solution

Isolate one of the terms with an absolute value sign.

$$|x-1| \geq 5+|x-3|$$

Remove the absolute value sign by splitting the inequality into two.

$$x-1 \geq (5+|x-3|) \quad \text{or} \quad x-1 \leq -(5+|x-3|)$$

Solve for the remaining term with an absolute value sign in each inequality.

$$|x-3| \leq x-6 \quad \text{or} \quad |x-3| \leq -x-4$$

Remove the absolute value signs by splitting each inequality into two.

$$\Big[-(x-6) \leq x-3 \leq x-6\Big] \quad \text{or} \quad \Big[-(-x-4) \leq x-3 \leq -x-4\Big]$$

$$\Big(x-3 \geq -x+6 \quad \text{and} \quad x-3 \leq x-6\Big) \quad \text{or} \quad \Big(x-3 \geq x+4 \quad \text{and} \quad x-3 \leq -x-4\Big)$$

Solve for x.

$$\Big(x \geq \frac{9}{2} \quad \text{and} \quad -3 \leq -6\Big) \quad \text{or} \quad \Big(-3 \geq 4 \quad \text{and} \quad x \leq -\frac{1}{2}\Big)$$

This is a false statement regardless of what x is. Therefore, there's no solution. This is reflected in the graph by the fact that the blue curve never reaches the red curve.